Centrifugal and axial compressors – Useful equations

Work done on the fluid per unit mass

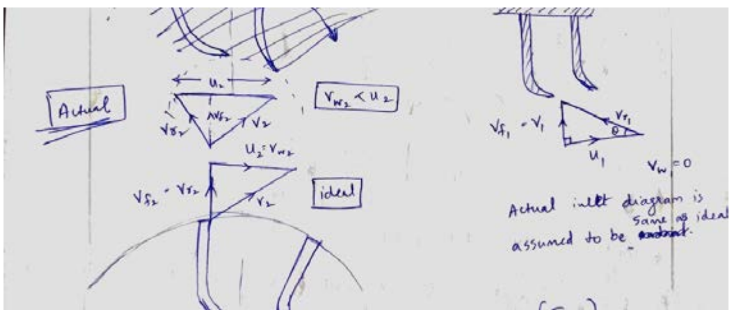
$$\frac{W}{m} = (V_{w2}U_2 - V_{w1}U_1)\frac{\mathrm{J}}{\mathrm{kg}}, \quad Euler's\ equation$$

For Radial Vanes (commonly used vane in centrifugal compressor), $V_{w1} = 0, V_{w2} = U_{2,}$

$$\frac{W}{m} = U_2{}^2$$

$$\frac{W}{m} = \left(\frac{V_2{}^2 - V_1{}^2}{2}\right) + \left(\frac{U_2{}^2 - U_1{}^2}{2}\right) + \left(\frac{V_{r1}{}^2 - V_{r2}{}^2}{2}\right), \text{General equation}$$

Velocity Triangle for radial flow vanes



Stagnation Enthalpy (h_{01})

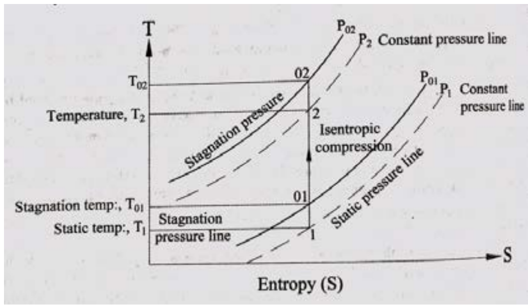
$$h_{01} = h_1 + \frac{V_1{}^2}{2}$$

Stagnation Temperature, $T_{01} = T_1 + \frac{V_1{}^2}{2C_p}$

Work done per kg of air based on SFEE , $\frac{W}{m} = h_{02} - h_{01} = C_p(T_{02} - T_{01})$

Or $\frac{W}{m} = C_pT_{01}\left(\frac{T_{02}}{T_{01}} - 1\right) = C_pT_{01}\left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right], \quad \frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$, 01 and 02 refers to stagnation states when the process is isentropic.

$$if\ V_1 = V_2, \frac{W}{m} = (h_2 - h_1) = C_p(T_2 - T_1) = C_pT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$



Slip factor ($\emptyset_s$)

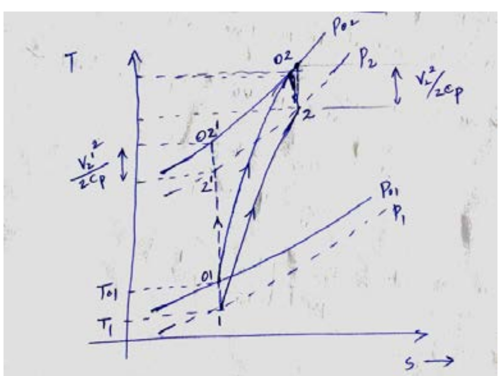
$$\emptyset_s = \frac{V_{w2}}{U_2}$$

when slip is considered, Euler work, $\frac{W}{m} = \emptyset_s \, {U_2}^2$

Work factor or power input factor, $\psi = \frac{actual\ work}{Euler\ work}$

$$\boldsymbol{C_p(T_{02} - T_{01}) = \psi\emptyset_s\, {U_2}^2}, if\ \psi\ is\ not\ given\ then\ RHS\ becomes\ \emptyset_s\, {U_2}^2$$

Comment [T1]: For finding out the impeller diamensions



$$Isentropic\ efficiency, \eta_{ise} = \frac{isentropic\ work}{Actual\ work} = \frac{isentropic\ temp\ rise}{Actual\ temp\ rise}$$

$$\eta_{ise} = \frac{C_p\left({T_{02}}' - T_{01}\right)}{C_p\left(T_{02} - T_{01}\right)}$$

$if\ V_1 = V_2$

$$\eta_{ise} = \frac{C_p\left(T_2{}' - T_1\right)}{C_p\left(T_2 - T_1\right)}$$

$$Stagnation\ Pressure\ ratio\ \frac{P_{02}}{P_{01}} = \left(\frac{T_{02}{}'}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$Static\ pressure\ ratio\ \frac{P_2}{P_1} = \left(\frac{T_2{}'}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$Width\ of\ impeller\ blades, at\ inlet\ B_1 = \frac{mass\ flow\ rate}{\rho_1\left(\pi D_1 V_{f1}\right)}$$

$$at\ outlet\ B_2 = \frac{mass\ flow\ rate}{\rho_2\left(\pi D_2 V_{f2}\right)}$$

Blade width at the Inlet of diffuser and outlet of impeller are same.

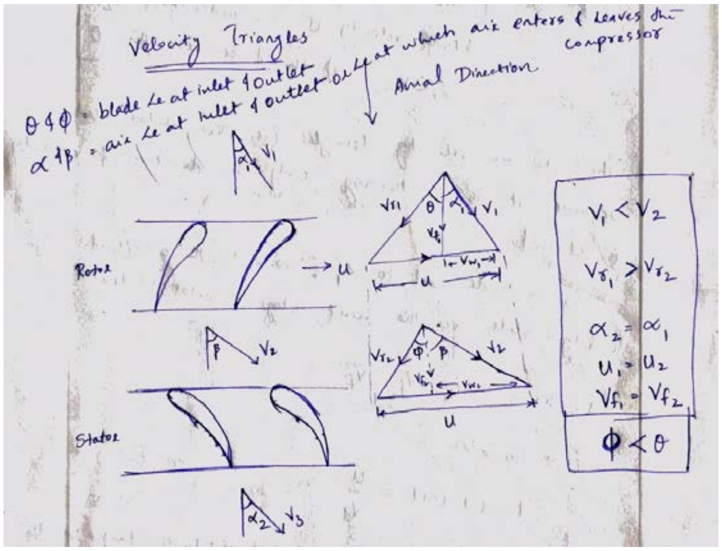
$If\ n\ is\ the\ number\ of\ blades\ and\ t\ is\ the\ thickness\ of\ each\ blade,$

$$mass\ flow\ rate = \rho_1(\pi D_1 - nt)B_1 V_{f1}$$

$$Pressure\ coefficient, \phi_{p=}\ \frac{isentropic\ work}{Euler\ work}$$

$$\phi_{p=}\ \psi\phi_s\eta_{ise}$$

Axial Flow Compressor



$tan\ \alpha + \tan\theta = \frac{U}{V_f}$

$tan\ \beta + \tan\phi = \frac{U}{V_f}$

$\boldsymbol{Work\ input\ per\ stage\ per\ kg\ of\ air},\frac{W}{m} = (V_{w2} - V_{w1})U$

$For\ finding\ out\ power\ required\ = (V_{w2} - V_{w1})U * \frac{N}{60}\ Watts$

$\boldsymbol{Total\ Work\ input\ per\ \ kg\ of\ air},\frac{W}{m} = (V_{w2} - V_{w1})U * \boldsymbol{no\ of\ stages} =\ C_p\left(T_{0f} - T_{0i}\right)$

Comment [T2]: For finding out the no of stages

$T_{0f}\ = Final\ stagnation\ temperature, T_{0i}\ = initial\ stagnation\ temperature$

$V_{w2}\ =\ V_{f2}\tan\beta,\ \ V_{w1}\ =\ V_{f1}\ tan\ \ \alpha$

$\frac{W}{m} = (V_{w2} - V_{w1})U = UV_f(\tan\beta - \tan\alpha)$

$$\eta_{ise} = \frac{C_p(T_{02}{}' - T_{01})}{C_p(T_{02} - T_{01})} \ or \ \eta_{ise} = \frac{C_p(T_2{}' - T_1)}{C_p(T_2 - T_1)}$$

Comment [T3]: For finding out temp difference

$$\boldsymbol{Degree\ of\ Reaction}, \Omega = \frac{pressure\ rise\ in\ the\ rotor}{Total\ pressure\ rise\ per\ stage}$$

$$\Omega = \frac{V_f}{2U}(\tan\theta + \tan\phi)$$

When $\Omega = 50\%$ or 0.5, $\mathbf{U} = \boldsymbol{V_f}\,(\mathbf{tan}\,\boldsymbol{\theta} + \mathbf{tan}\,\boldsymbol{\phi})$

And when $\Omega = 50\%$ or 0.5, $\boldsymbol{\alpha} = \boldsymbol{\phi}$ **and** $\boldsymbol{\beta} = \boldsymbol{\theta}$, known as symmetric blading